Logic gates

→ Two types of logic gates
- Basic logic gates (AND, OR, NOT)
- Universal logic gates (NAND, NOR]

→ Additional logic gates
- EX-OR
- EX-NOR

Def: Logic gates are fundamental building blocks of digital systems

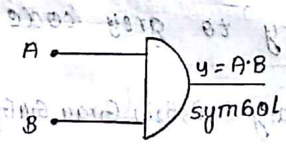
NAND → NOT of AND

NOR → NOT of OR

The interconnection of gates to perform a variety of logical operations is called logic design

Basic gates

i) AND gate



symbol

Truth Table

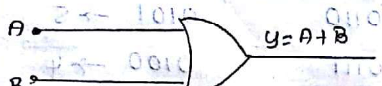
A	B	Y = A·B
0	0	0
0	1	0
1	0	0
1	1	1

Ex:- 2 i/p AND gate

Other names → all or nothing gate

IC 7408 → four 2 i/p AND gate are available

OR gate

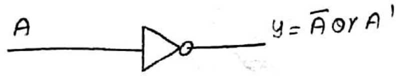


Truth Table

A	B	y = A+B
0	0	0
0	1	1
1	0	1
1	1	1

7432 - four 2 i/p gates

Not gate (inverter)

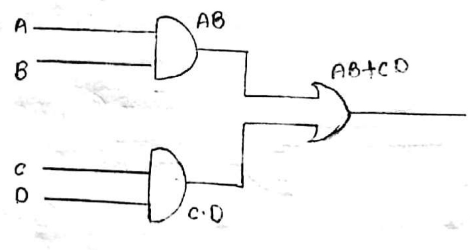


Truth Table

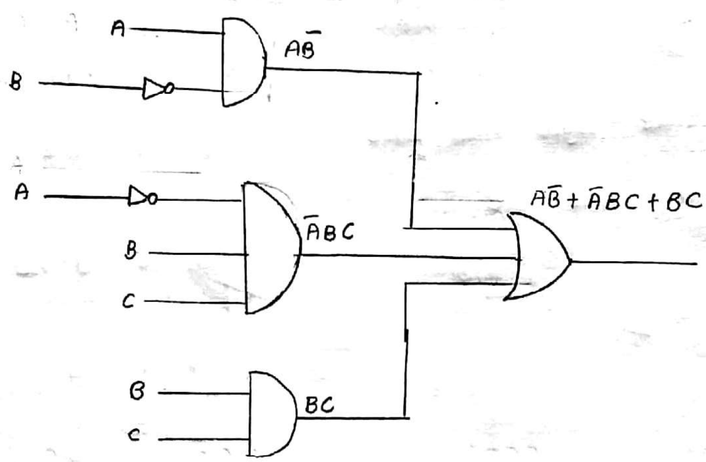
A	$y = \bar{A}$
0	1
1	0

7404 - six not gates

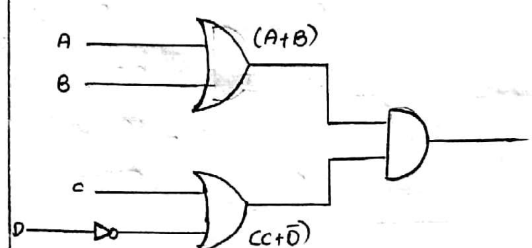
Draw the circuit for the given expression $F = AB + CD$



Draw the circuit for given expression $F = A\bar{B} + \bar{A}BC + BC$

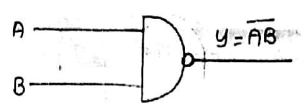


$F = (A+B) \cdot (C+\bar{D})$



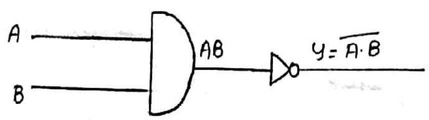
Universal logic gates:

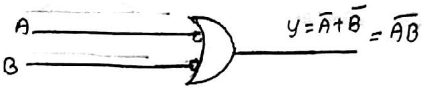
NAND gate:



Truth table

A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0





It is also called as bubbled OR gate

+ve true logic NAND gate is equivalent to −ve logic 'OR' gate

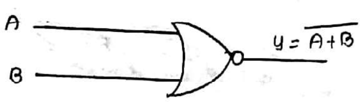
NOR gate

⇒ −ve true logic AND gate

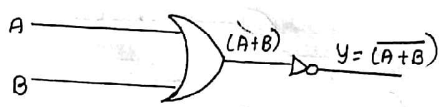
⇒ Bubbled AND gate

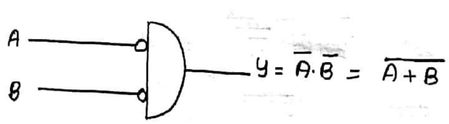
Truth Table

A	B	$y = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0



(or)



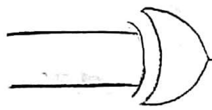


Demorgan's law:

$\overline{A+B} = \bar{A}\cdot\bar{B}$

$\overline{A\cdot B} = \bar{A}+\bar{B}$

Ex-OR gate:



$Y = A \oplus B$

(or)

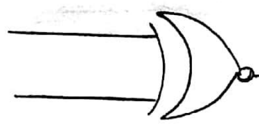
$\bar{A}B + A\bar{B}$

Truth Table

A	B	$Y = A \oplus B$
0	0	0
0	1	1
0	1	1
1	1	0

Y = '1' when there is odd no. of 1's in input otherwise '0'

Ex-NOR gate



$Y = A \odot B$

$= AB + \bar{A}\bar{B}$

Truth Table

A	B	$Y = A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1

Bubbled OR gate → NAND Gate

Bubbled NAND gate → ~~Bubbled~~ OR gate

Bubbled AND gate → NOR gate

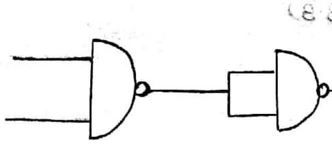
Bubbled NOR gate → AND gate

Realisation of logic gates

1. using NAND gate:

(1) AND gate

$Y = A \cdot B$

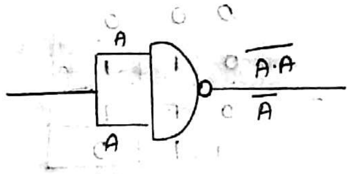


$Y = \overline{\overline{A \cdot B}}$

$= A \cdot B$

2) NOT gate:

$y = \overline{A}$

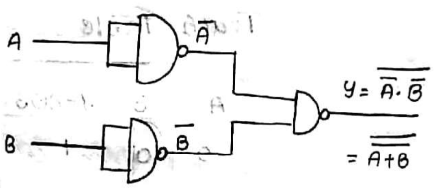


3) OR gate:

$y = A+B$

$= \overline{\overline{A+B}}$

$= \overline{\overline{A} \cdot \overline{B}}$

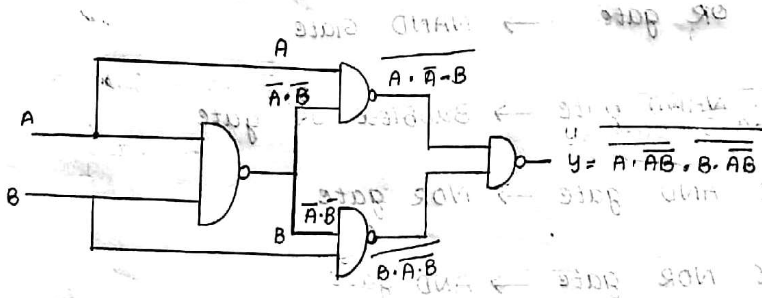


$= \overline{\overline{A+B}}$

$= A+B$

4. EX-OR gate

$y = \overline{A}B + A\overline{B}$



$y = \overline{\overline{A \cdot \overline{AB}} \cdot \overline{B \cdot \overline{AB}}}$

$= \left(\overline{\overline{A \cdot \overline{A \cdot B}}}\right) + \left(\overline{\overline{B \cdot \overline{A \cdot B}}}\right)$

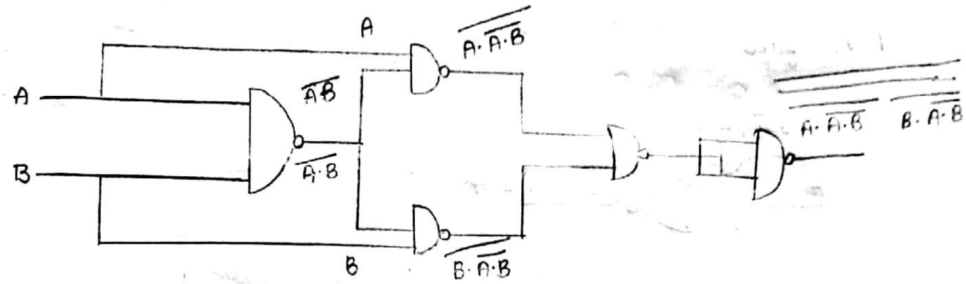
$= (A \cdot \overline{A \cdot B}) + (B \cdot \overline{A \cdot B})$

$= \left[(A \cdot (\overline{A}+\overline{B})\right] + \left[(B \cdot (\overline{A}+\overline{B}))\right]$

$= (A \cdot \overline{A}) + (A\overline{B}) + (\overline{A} \cdot B) + (B \cdot \overline{B})$

$= A\overline{B} + \overline{A}B = A \oplus B$

Ex-NOR gate



$$y = \overline{\overline{\overline{A\cdot\overline{A\cdot B}}\cdot\overline{B\cdot\overline{A\cdot B}}}}$$

$$= \overline{A\cdot\overline{A\cdot B}}\cdot\overline{B\cdot\overline{A\cdot B}}$$

$$= \overline{A\cdot(\overline{A}+\overline{B})}\cdot\overline{B\cdot(\overline{A}+\overline{B})}$$

$$= \overline{A}+(\overline{\overline{A}+\overline{B}})\cdot\overline{B}+(\overline{\overline{A}+\overline{B}})$$

$$= [\overline{A}+\overline{\overline{A}}\cdot\overline{\overline{B}}]\cdot[\overline{B}+(\overline{\overline{A}}\cdot\overline{\overline{B}})]$$

$$= [\overline{A}+AB]\,[\overline{B}+(AB)]$$

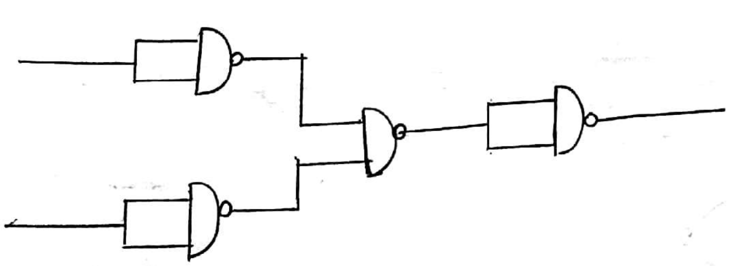
$$= \overline{A}\,\overline{B}+\underbrace{\overline{A}AB}_{0}+\underbrace{AB\overline{B}}_{0}+AB\,AB$$

$$y = \overline{A}\,\overline{B}+AB \qquad A\cdot\overline{A}=0$$

$$y = A\odot B \qquad A\cdot A = A$$

Boolean laws

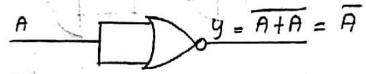
NOR gate



NOR gate

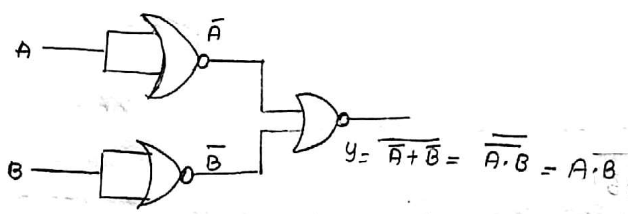
1. NOT gate

$y = \overline{A}$



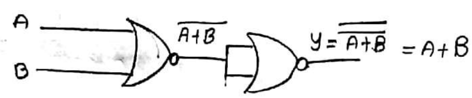
2. AND gate

$y = A\cdot B = \overline{\overline{A \cdot B}} = \overline{\overline{A} + \overline{B}}$



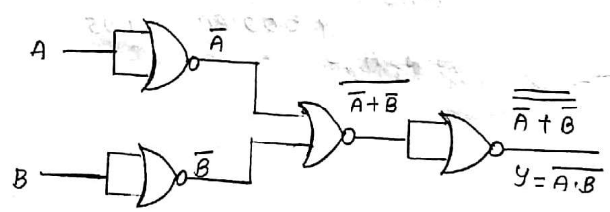
3. OR gate

$y = A+B = \overline{\overline{A+B}}$

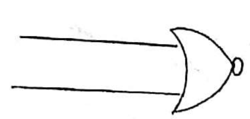


4. NAND gate

$y = \overline{A \cdot B} = \overline{\overline{\overline{A} + \overline{B}}}$



5. EX-OR gate



NAND- NAND and NOR-NOR implementation

I) using NAND

1. Draw the circuit using basic gates

2. If NAND hardware has been choosen add bubbles at the output of each AND gate and add bubbles at the inputs of OR gate

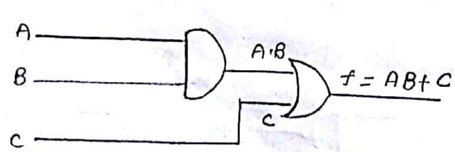
3. If NOR hardware has been choosen add bubbles at the output of OR gate and add bubbles at the inputs of each AND gate.

4. Add an invertor (NOT gate) on each line receive a bubble in step 2 and 3.

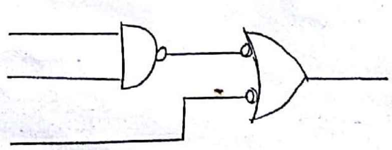
5. Replace bubbled OR gate by NAND gate and bubbled AND gate by NOR gate.

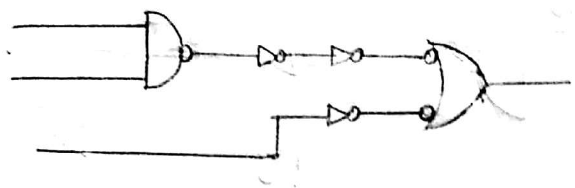
6. Eliminate double invertions.

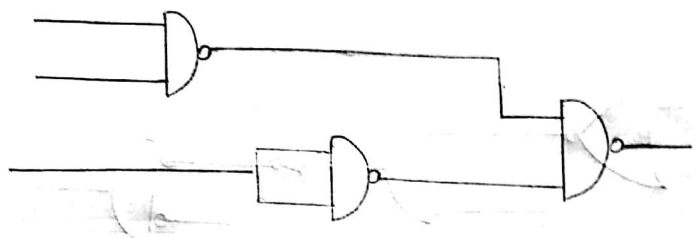
Eg:- $f = AB + C$



Basic gates



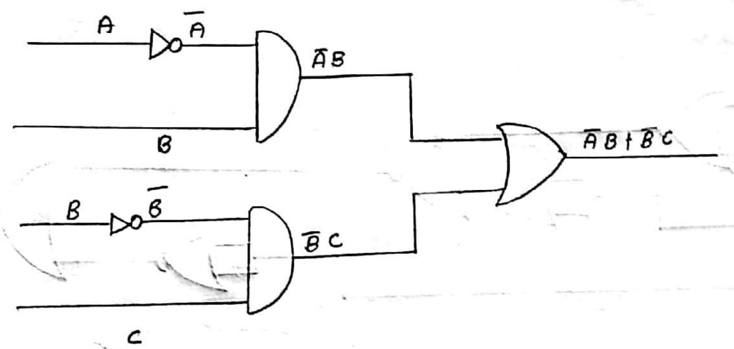


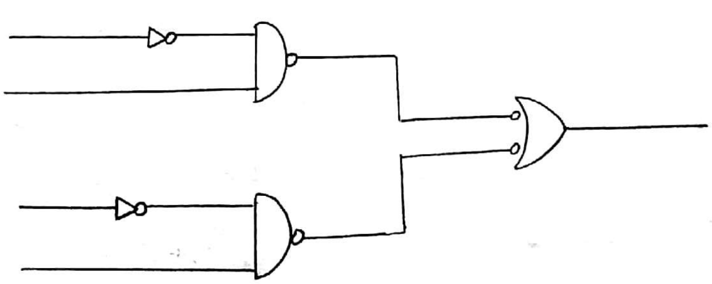


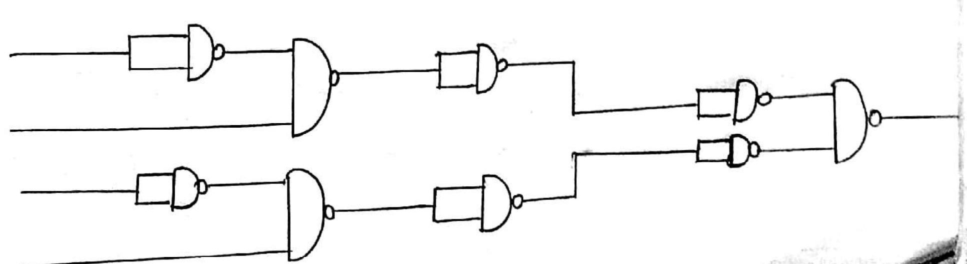
f= ~~$\overline{(A+B)\cdot C\cdot D}$~~ $f = \overline{(A+B)\cdot C}\cdot D$ using only NOR gates

$f = \bar{A}B+\bar{B}C$ using only NAND gates

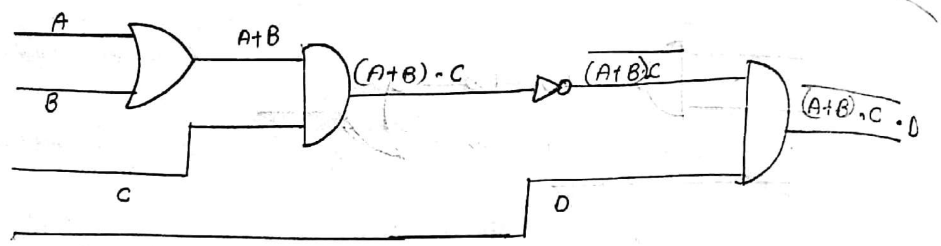
ii)

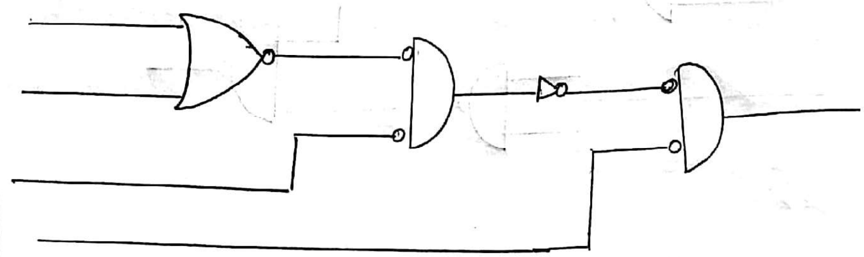


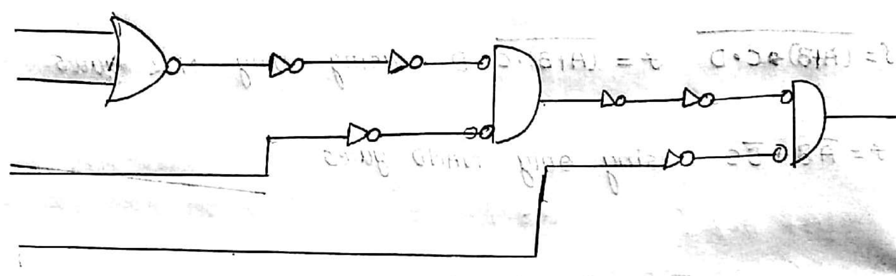


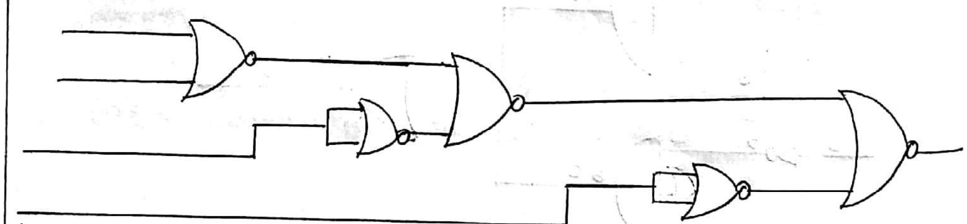


i)









Boolean algebra and Minimization techniques

Concepts

→ Boolean theorems

→ Simplification or minimization using Boolean algebra

→ standard SOP & POS

→ Karnaugh maps

→ 2, 3, 4, 5, 6 – variable k-maps.

Boolean algebra is a system of mathematical logic. It is a algebraic system consisting of set of elements (0, 1) with two binary operators 'OR' & 'AND' and unary operator 'NOT'.

It is a basic mathematical tool in the analysis and synthesis of switching circuits.

It is a way to express logic functions algebrically

Any complex logic statements can be expressed by a boolean function

Boolean algebra

Boolean laws and theorems

1. AND laws

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot \bar{A} = 0$

$A \cdot A = A$

2. OR laws

$A + 0 = A$

$A + 1 = \cancel{A}1$

$A + \bar{A} = 1$

$A + A = A$

3. Absorption laws

$A + AB = A$

$A(A+B) = A$

4. Redundant laws

$A + \bar{A}B = A+B$

$A(\bar{A}+B) = AB$

5. Commutative law

$A+B = B+A$

$AB = BA$

6. Distributive law

$A(B+C) = AB + AC$

$A + (BC) = (A+B)(A+C)$

7. Associative law

$(A+B)+C = A+(B+C)$

$(AB)C = A(BC)$

8. Demorgan's law:

$\overline{A+B} = \bar{A} \cdot \bar{B}$

$\overline{A \cdot B} = \bar{A} + \bar{B}$

Ex:- Prove $A(B+C) = AB + AC$

A	B	C	B+C	A·(B+C)	AB	AC	AB+AC
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Simplify the following expansion

1. $x(x+yz)$

By absorption law

$x(x+y+z) = x$

2. $AB+BC+C$

$= AB+C(B+1)$

$= AB+C(1)$

$= AB+C$

3. $ABC+AB\bar{C}+A\bar{B}C$

$= A(BC+B\bar{C}+\bar{B}C)$

$= A[B(C+\bar{C})+\bar{B}C]$

$= A(B+\bar{B}C)$

$= A(B+C)$

4. $\overline{ABC}+\overline{A}\overline{B}C+A\overline{BC}$

$= \overline{AB}+\bar{C}+\overline{AB}C+A\overline{BC}$

$= \overline{AB}(1+C)+\bar{C}+A\overline{BC}$

$= \overline{AB}(1)+\bar{C}+A\overline{BC}$

$= \overline{AB}+\bar{C}+A(\bar{B}+\bar{C})$

$= \overline{AB}+A\bar{B}+A\bar{C}$

$= \overline{AB}+A\bar{B}+\bar{C}$

$= \bar{A}+\bar{B}+A\bar{B}+\bar{C}$

$= \bar{A}+\bar{B}+\bar{C}$

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4. $A+B+C+D+E+1$

$= 1$

5. $y = A \cdot \bar{A} C$

$= 0$

6. $x\bar{y}\bar{z} + \bar{y}\bar{z} + x$

$= \bar{y}\bar{z}(x+1) + x$

$= \bar{y}\bar{z} + x$

7. $A(A+B)$

$= AA + AB$

$= A + AB$

$= A(1)$

$= A$

8. $AB + \bar{A}B + BC$

$= B(A+\bar{A}) + BC$

$= B + BC = B(1+C) = B$

Boolean theorems

1. $AB + \bar{A}C + BC = AB + \bar{A}C$ [consensus theorem]

$AB + \bar{A}C + BC = AB + \bar{A}C + [BC \cdot 1]$

$= AB + \bar{A}C + BC[A+\bar{A}]$

$= AB + \bar{A}C + ABC + \bar{A}BC$

$= AB(1+C) + \bar{A}C(1+B)$

$= AB + \bar{A}C$

2. $(A+B) \cdot (\bar{A}+C) \cdot (B+C) = (A+B)(\bar{A}+C)$

$(A+B)(\bar{A}+C)(B+C) = (A\bar{A} + AC + \bar{A}B + BC)(B+C)$

$= ABC + AC + \bar{A}B + \bar{A}BC + BC + BC$

$= BC(A + \bar{A} + 1 + 1) + AC + \bar{A}B$

$= BC + AC + \bar{A}B$

$(A+B)(\bar{A}+C) = A\bar{A} + AC + \bar{A}B + BC$

$= AC + \bar{A}B + BC$

$\therefore LHS = RHS$

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Transposition theorem

1. $AB+\bar{A}C = (A+C)(\bar{A}+B)$

$= A\bar{A} + AB + \bar{A}C + BC$

$= AB + \bar{A}C + BC$

$= AB+\bar{A}C + [BC \cdot 1]$

$= AB+\bar{A}C + BC[A+\bar{A}] = AB+\bar{A}C + ABC + \bar{A}BC$

$= AB[1+C] + \bar{A}C[1+B]$

$= AB + \bar{A}C$

Minimise the given Boolean expansion

1. $f = AB + \bar{A}B + \bar{A}\cdot\bar{B}$

$= AB + \bar{A}(B+\bar{B})$

$= AB + \bar{A}$

$= \bar{A} + B$

2. $f = A[B+\bar{C}(\overline{\bar{A}B+A\bar{C}})]$

$= A[B+\bar{C}(\overline{\bar{A}B} \cdot \overline{A\bar{C}})]$

$= A[B+\bar{C}[(\bar{\bar{A}}+\bar{B})\cdot(\bar{A}+C)]$

$= A[B+\bar{C}[\bar{A}C + \bar{B}\bar{A} + \bar{B}C]]$

$= A[B+\bar{A}\bar{B}\bar{C}] = AB$

3. $f = \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + ABC + \bar{A}\bar{B}\bar{C}$

$= \bar{B}C(\bar{A}+A) + BC(\bar{A}+A) + \bar{A}\bar{B}C$

$= \bar{B}C + BC + \bar{A}\bar{B}\bar{C}$

$= \bar{B}(C+\bar{A}\bar{C}) + BC = \bar{B}(C+\bar{A}) + BC$

$= \bar{B}C + \bar{A}\bar{B} + BC = \bar{A}\bar{B} + C$

4. $f=(x+y)(\bar{x}\bar{z}+z)(\bar{y}+xz)$

$=(xz+y\bar{x}\bar{z}+yz)(\bar{y}+xz)$

$=x\bar{y}z+xz+xyz$

$=x\bar{y}z+xz(1+y)$

$=xz(1+\bar{y})$

$=xz$

ABC			Max terms (M_i) ↓ 0	min terms (m_i) ↓ 1
0	0	0	$A+B+C \rightarrow M_0$	$\bar{A}\cdot\bar{B}\cdot\bar{C} \rightarrow m_0$
0	0	1	$A+B+\bar{C} \rightarrow M_1$	$\bar{A}\cdot\bar{B}\cdot C \rightarrow m_1$
0	1	0	$A+\bar{B}+C \rightarrow M_2$	$\bar{A}\cdot B\cdot\bar{C} \rightarrow m_2$
0	1	1	$A+\bar{B}+\bar{C} \rightarrow M_3$	$\bar{A}\cdot B\cdot C \rightarrow m_3$
1	0	0	$\bar{A}+B+C \rightarrow M_4$	$A\cdot\bar{B}\cdot\bar{C} \rightarrow m_4$
1	0	1	$\bar{A}+B+\bar{C} \rightarrow M_5$	$A\cdot\bar{B}\cdot C \rightarrow m_5$
1	1	0	$\bar{A}+\bar{B}+C \rightarrow M_6$	$AB\bar{C} \rightarrow m_6$
1	1	1	$A+B+C \rightarrow M_7$	$\bar{A}\bar{B}\bar{C} \rightarrow m_7$
Note:	A		0	1
	$\bar{A}$		1	0

Eg:- $f=\bar{A}\bar{B}+\bar{A}B$ (sum of products) can also be expressed as $(\bar{A}+B)(\bar{A}+\bar{B})$ (Product of sums)

	A	B	$f=\bar{A}\bar{B}+\bar{A}B$
$\bar{A}\bar{B}$	0	0	1
$\bar{A}B$	0	1	1
$A\bar{B}$	1	0	0
AB	1	1	0

$f(A,B)=\bar{A}\bar{B}+\bar{A}B$

$=m_0+m_1$

$=\Sigma m(0,1)$

$f(A,B) = (\bar{A}+B)\cdot(\bar{A}+\bar{B})$

$= M_2 \cdot M_3$

$= \Pi M(2,3)$

→ Converting normal SOP into standard SOP

1. Find out missing literal

2. AND (·) the missing literal '+' its complement

3. Expand the terms and reorder the literals

4. Neglect the repeated terms

Eg:- $f = AB + \bar{B}C$

$f = AB\cdot(C+\bar{C}) + \bar{B}C\cdot(A+\bar{A})$

$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

$= 111 + 110 + 101 + 001$

$= m_7 + m_6 + m_5 + m_1$

$= \Sigma m(7,6,5,1) = \Sigma m(1,5,6,7)$

ii $f(A,B) = \bar{A} + \bar{B}$

$= \bar{A}\cdot(B+\bar{B}) + \bar{B}\cdot(A+\bar{A})$

$= \bar{A}\cdot B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B}$

$= \bar{A}\cdot B + A\cdot\bar{B} + \bar{A}\bar{B}$

$= 01 + 10 + 00$

$= m_1 + m_2 + m_0$

$= \Sigma m(0,1,2)$

iii) $f(A,B,C) = \bar{A}B + \bar{A}BC + A\bar{B} + A\bar{B}C$

$= \bar{A}B.(C+\bar{C}) + \bar{A}BC + A\bar{B}.(C+\bar{C}) + A\bar{B}C$

$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$$

$$= \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

$$= 011 + 010 + 101 + 100$$

$$= m_3 + m_2 + m_5 + m_4$$

$$= \Sigma m(2,3,4,5)$$

$$f(A,B,C,D) = AB + BCD + \bar{B}\bar{C}\bar{D}$$

$$= AB\cdot(\bar{C}+C)\cdot(\bar{D}+D) + BCD(\bar{A}+A) + \bar{B}\bar{C}\bar{D}(\bar{A}+A)$$

$$= AB\cdot[\bar{C}\bar{D}+\bar{C}D+C\bar{D}+CD] + BCD\bar{A} + ABCD + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$= AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD + \bar{A}BCD + ABCD + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$= AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + \bar{A}BCD + ABCD + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$= 1100 + 1101 + 1110 + 0111 + 1111 + 1000 + 0000$$

$$= m_{12} + m_{13} + m_{14} + m_7 + m_{15} + m_8 + m_0$$

$$= \Sigma m(0,7,8,12,13,14,15)$$

$$f(A,B,C) = \bar{A} + BC$$

$$= \bar{A}\cdot(B+\bar{B})\cdot(C+\bar{C}) + BC(A+\bar{A})$$

$$= \bar{A}\cdot(BC + B\bar{C} + \bar{B}C + \bar{B}\bar{C}) + BCA + BC\bar{A}$$

$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ABC + \bar{A}BC$$

$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ABC$$

$$= 011 + 010 + 001 + 000 + 111$$

$$= m_3 + m_2 + m_1 + m_0 + m_7$$

$$= \cancel{m}\,\Sigma m(0,1,2,3,7)$$

Converting normal POS into standard POS

1. Find out the missing literal

→ OR(+) the sum-term with the missing literal .(·) its complement

→ Expand the terms using distributive law and reorder the literals

→ neglect the repeated terms

$f = (A+B)(\bar{B}+\bar{C})$

$= [(A+B)+[C\cdot\bar{C}]]\cdot[(\bar{B}+\bar{C})+(A\cdot\bar{A})]$

$= (A+B+C)(A+B+\bar{C})\cdot(\bar{B}+\bar{C}+A)(\bar{B}+\bar{C}+\bar{A})$

$= 000 \quad 001 \quad 010 \quad 110$

$= \Sigma\Pi M(0,1,2,6)$

$f(A,B,C) = (A+\bar{B}+C)(A+\bar{B})(\bar{B}+\bar{C})$

$= (A+\bar{B}+C)[(A+\bar{B})+(C\bar{C})]\cdot[(\bar{B}+\bar{C})+(A\bar{A})]$

$= (A+\bar{B}+C)[A+\bar{B}+C)(A+\bar{B}+\bar{C})\cdot(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

$= (010)\ (011)\ (111)$

$= \Sigma\Pi M(2,3,7)$

$f(A,B,C,D) = (A+B+C+\bar{D})(A+\bar{B}+C)$

$= (A+B+C+\bar{D})[(A+\bar{B}+C)+(D\bar{D})]$

$= (A+B+C+\bar{D})[A+\bar{B}+C+D][A+\bar{B}+C+\bar{D}]$

$= (0001)\ (0100)\ (0101)$

$= \Sigma\Pi M(1,4,5)$

Expand the function output $f = A\bar{B}C + \bar{A}\bar{B} + \bar{B}\bar{C}$ to min terms and max terms

$$f = A\bar{B}C + \bar{A}\bar{B} + \bar{B}\bar{C}$$

$$= A\bar{B}C + \bar{A}\bar{B}\cdot(C+\bar{C}) + \bar{B}\bar{C}(A+\bar{A})$$

$$= A\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$= A\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$$= (101) + (001) + (000) + (100)$$

$$= \Sigma m(0,1,4,5)$$

$$= \Pi M(2,3,6,7)$$

$$f(AB) = (\bar{A})(\bar{B})$$

$$= [(\bar{A}) + (\bar{B}\cdot B)][(\bar{B}) + (A\bar{A})]$$

$$= (\bar{A}+\bar{B})(\bar{A}+B)(\bar{B}+A)(\bar{B}+\bar{A})$$

$$= (\bar{A}+\bar{B})(\bar{A}+B)(\bar{B}+A)$$

$$= (11)(10)(01)$$

$$= \Pi M(1,2,3)$$

$$f(A,B,C,D) = (A+B+C+\bar{D})(A+B+C)$$

$$= (A+B+C+\bar{D})[(A+B+C)+(D\cdot\bar{D})]$$

$$= (A+B+C+\bar{D})[A+B+C+D][A+B+C+\bar{D}]$$

$$= (A+B+C+D)(A+B+C+\bar{D})$$

$$= (0000)(0001)$$

$$= \Pi M(0,1)$$

Duality and complements of Boolean expression

duality	complement
AND(·) $\longrightarrow$ OR(+)	AND(·) $\longrightarrow$ OR(+)
OR(+) $\longrightarrow$ AND(·)	OR(+) $\longrightarrow$ AND(·)
$0 \longrightarrow 1$	$0 \longrightarrow 1$
$1 \rightarrow 0$	$1 \rightarrow 1$
	$A \rightarrow \bar{A}$
	$\bar{A} \rightarrow A$

Find the duality and complement of given function

$$f = (\bar{A}B) + (\bar{B}\cdot C) + (ABC)$$

Duality

$$F = (\bar{A}+B)\cdot(\bar{B}+C) + (A+B+C)$$

Complement

$$f = (A+\bar{B})\cdot(B+\bar{C})\cdot(\bar{A}+\bar{B}+\bar{C})$$

Eg:-

duality $0\cdot 1 = 0$

$0+1 = 1$

Karnaugh map (k-map):-

2, 3, 4, 5, 6 variable k-map

A \ B	$\bar{B}$	B
$\bar{A}$	0	1
A	2	3

2 variable k-map

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	3	2
A	4	5	7	6

3-variable k-map

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

Eg:- $f(A,B) = \bar{A}B + \bar{A}\bar{B} + A\bar{B}$

$= \bar{A}(B+\bar{B}) + A\bar{B}$

$= \bar{A} + A\bar{B}$

$= \bar{A} + \bar{B}$

$A \backslash B$	$\bar{B}$	B
$\bar{A}$	1	1
A	1	0

$f = \bar{B}(\bar{A}+A) + \cancel{\bar{A}B}\ \bar{A}(\bar{B}+B)$

$= \bar{B} + \bar{A}(\bar{B}+B)$

$= \bar{A} + \bar{B}$

$f = \bar{A}B + AB + A\bar{B}$

$= B(A+\bar{A}) + A\bar{B}$

$= B + A$

$A \backslash B$	$\bar{B}$	B
$\bar{A}$	0	1
A	1	1

$f = B(\bar{A}+A) + A(\bar{B}+B)$

$= A + B$

$f = \Sigma m(0, 1, 6, 7)$ Reduce using k-map

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$ ← pair	1 (0)	1 (1)	0 (3)	0 (2)
A	0 (4)	0 (5)	1 (7)	1 (6) → pair

$f = \bar{A}(\bar{B}\bar{C} + \bar{B}C) + A(BC + B\bar{C})$

$= \bar{A}(\bar{B}(\bar{C}+C)) + A(B(C+\bar{C}))$

$= \bar{A}\bar{B} + AB$

$f(A,B,C) = \Sigma m(0,1,4,5)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	1	0	0
A	1	1	0	0

$f = (\bar{A}+A)\cdot(\bar{B}\bar{C}+\bar{B}C)$

$= 1\cdot \bar{B}(\bar{C}+C)$

$= \bar{B}$

$f(A,B,C) = \Sigma m(0,1,5,7)$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	1	0	0
A	0	1	1	0

$f = \bar{A}(\bar{B}\bar{C}+\bar{B}C) + A(\bar{B}C+BC)$

$= \bar{A}(\bar{B}(\bar{C}+C)) + A(C(\bar{B}+B))]$

$= \bar{A}\bar{B}+AC$

$f(A,B,C,D) = \Sigma m(1,3,5,7)\ 12,14)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	1	0
$\bar{A}B$	0	1	1	0
AB	1	0	0	1
$A\bar{B}$	0	0	0	0

$f = (\bar{A}\bar{B}+\bar{A}B)\cdot(\bar{C}D+CD) + AB(\bar{C}\bar{D}+C\bar{D})$

$= [\bar{A}(\bar{B}+B)]\cdot[D(\bar{C}+C)] + AB[\bar{D}(\bar{C}+C)]$

$= \bar{A}\cdot D + AB\bar{D}$

$= \bar{A}D + AB\bar{D}$

$f(A,B,C,D) = \Sigma m(0,2,3,8,10,11,12,14)$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1: 0	3: 1	2: 1
$\bar{A}B$	4: 0	5: 0	7: 0	6: 0
AB	12: 1	13: 0	15: 0	14: 1
$A\bar{B}$	8: 1	9: 0	11: 1	10: 1

$$(A\bar{B}+\bar{A}\bar{B})\cdot(CD+C\bar{D}) + (AB+A\bar{B})\cdot(\bar{C}\bar{D}+C\bar{D}) + (\bar{A}\bar{B}+A\bar{B})(\bar{C}\bar{D}+C\bar{D})$$

$$[\bar{B}(A+\bar{A})]\cdot[C(\bar{D}+D)] + [A(B+\bar{B})]\cdot[\bar{D}(\bar{C}+C)] + [\bar{B}(\bar{A}+A)](\bar{D}(\bar{C}+C))$$

$$\bar{B}C + A\bar{D} + \bar{B}\bar{D}$$

$f(A,B,C,D) = \Sigma m(0,1,2,3,9,11,12,13,14)$

K-map using don't care combinations

$f(A,B,C) = \Sigma m(1,3,5) + \Sigma d(7)$

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	1	0
A	0	1	X	1

$\Rightarrow$

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	1	0
A	0	1	1	0

X - '1' - pair, quad, or an octane

X - '0'

Quad $= (\bar{A}+A)\cdot(B\bar{C}+\bar{B}C)$
$= C$

Scanned by CamScanner

$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,4)$

AB \ CD	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	(0) X	(1) 1	(3) 1	(2) X
$\bar A B$	(4) X	(5) 0	(7) 1	(6) 0
AB	(12) 0	(13) 0	(15) 1	(14) 0
$A\bar B$	(8) 0	(9) 0	(11) 1	(10) 0

$\Longrightarrow$

AB \ CD	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	1	1	1	1
$\bar A B$	0	0	1	0
AB	0	0	1	0
$A\bar B$	0	0	1	0

$(\bar A\bar B + \bar A B + AB + A\bar B)\cdot CD + \bar A\bar B(\bar C\bar D + \bar C D + CD + C\bar D)$

$= (\bar A + A)\cdot CD + \bar A\bar B(\bar C + C)$

$= CD + \bar A\bar B$

$f(A,B,C,D) = \Sigma m(0,7,8,9,10,12) + \Sigma d(2,5,13)$

AB \ CD	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	(0) 1	(1) 0	(3) 0	(2) X
$\bar A B$	(4) 0	(5) X	(7) 1	(6) 0
AB	(12) 1	(13) X	(15) 0	(14) 0
$A\bar B$	(8) 1	(9) 1	(11) 1	(10) ϕ

AB \ CD	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	(0) 1	(1) 0	(3) 0	(2) 1
$\bar A B$	(4) 0	(5) 1	(7) 1	(6) 0
AB	(12) 1	(13) 1	(15) 0	(14) 0
$A\bar B$	(8) 1	(9) 1	(11) 0	(10) 1

$= (AB + A\bar B)\cdot(\bar C\bar D + \bar C D) + (\bar A\bar B + A\bar B)\cdot(\bar C\bar D + C\bar D)$
$\quad + \bar A B(\bar C D + CD)$

$= A\bar C + \bar B\bar D + \bar A B D$

$f(A,B,C,D) = \Pi M(0,3,4,7,8,10,12,14) + \Pi d(2,6)$

AB \ CD	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	(0) 0	(1) 1	(3) 0	(2) X
$\bar A B$	(4) 0	(5) 1	(7) 0	(6) X
AB	(12) 0	(13) 1	(15) 1	(14) 0
$A\bar B$	(8) 0	(9) 1	(11) 1	(10) 0

$\Longrightarrow$

(0) 0	(1) 1	(3) 0	(2) 0
(4) 0	(5) 1	(7) 0	(6) 0
(12) 0	(13) 1	(15) 1	(14) 0
(8) 0	(9) 1	(11) 1	(10) 0